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INELASTIC RESONANCES IN 1D AND 2D CONFINEMENT

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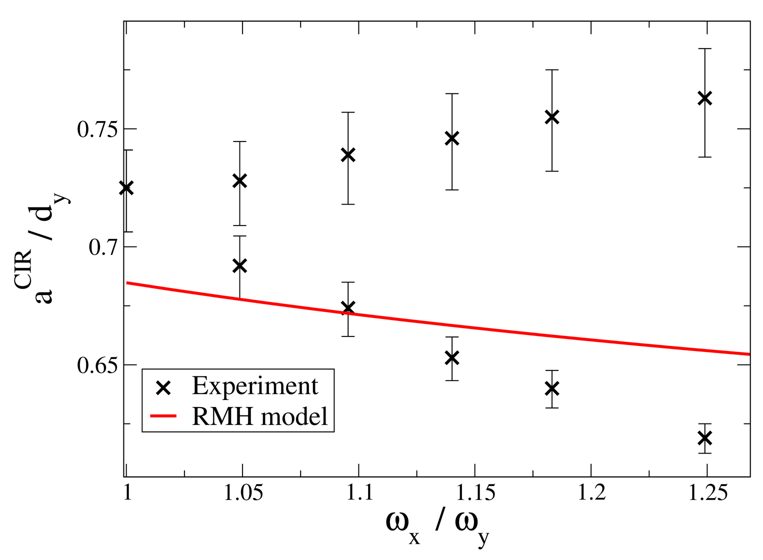
Motivation

The established theories of **confinement-induced resonances** (CIRs) [1,2,3] predict divergences of the effective interaction strength g for strong anisotropic confinement. These models are based on the relative motion Hamiltonian of two atoms in an harmonic trapping potential (RMH model). Experimentally, the divergence of g_{2D} was very recently demonstrated using rf spectroscopy [4]. However, an analysis of the RMH models [5,6] shows that they seem to contradict recent experimental observations of loss resonances in strongly anisotropic traps [7].

In this work [8], we present a model based on the coupling of relative and center-of-mass motion (CRC model) due to the anharmonicity of the external potential which yields (true) Feshbach resonances. We compare the resulting inelastic resonances to the loss resonances in [7].

Elastic vs. Loss Resonances in 1D

1D confinement is characterized by $\omega_x, \omega_y \gg \omega_z$. The divergence of the effective interaction strength g_{1D} predicted by the RMH model is an elastic scattering process which may not be visible in the loss experiment of Haller *et al.* [7]:



In the plot a comparison of the **elastic CIR** positions predicted by the RMH model (red line) with the **loss resonances** observed in the experiment [7] for different values of the anisotropy of the transversal confinement is shown. While the loss resonances show a **splitting** for transversal anisotropy, the elastic CIR does not split as proven in [5,6]. Hence, **the elastic RMH model is not capable of describing the loss resonances of the experiment [7]**.

⟹ Different behavior of elastic CIR and loss resonances in 1D.

Elastic vs. Loss Resonance in 2D

2D confinement is characterized by $\omega_x \gg \omega_y, \omega_z$. As in 1D, an elastic CIR is predicted by the divergence of the effective interaction strength g_{2D} by an RMH model [2]. This **elastic CIR occurs only for negative values of the scattering length** and was recently confirmed experimentally [4]. However, the experiment [7] observes a 2D loss resonance only at a positive value of the scattering length.

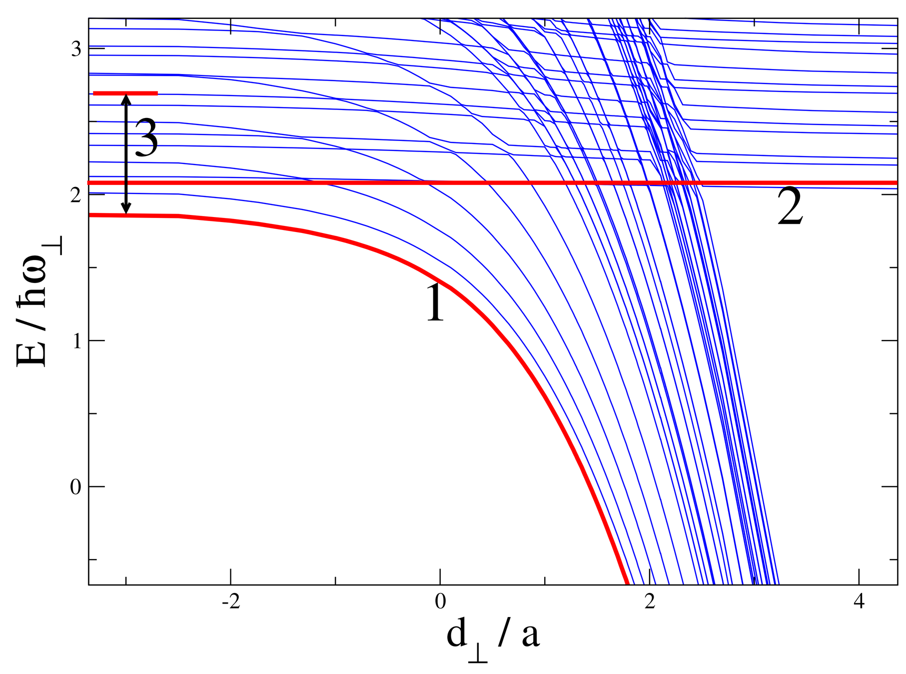
⟹ Different positions of elastic CIR and loss resonance in 2D.

Universal Model for Inelastic CIRs

Idea:

In the experiment [7] an optical lattice is used leading to a coupling of relative (REL) and center-of-mass (COM) motion. The COM-REL coupling leads to **Feshbach resonances** induced by (avoided) crossings of bound states with COM excitation and a state of an unbound atom pair without COM excitation. At the avoided crossing a molecular bound state can be occupied, since the excess binding energy can be transferred to COM excitation energy, which is a **inelastic** process. The **formation of molecules** is a loss mechanism and should be observable in loss experiments like [7]. The proposed model is **universally** valid for 1D as well as 2D confinement.

Energy Formula:

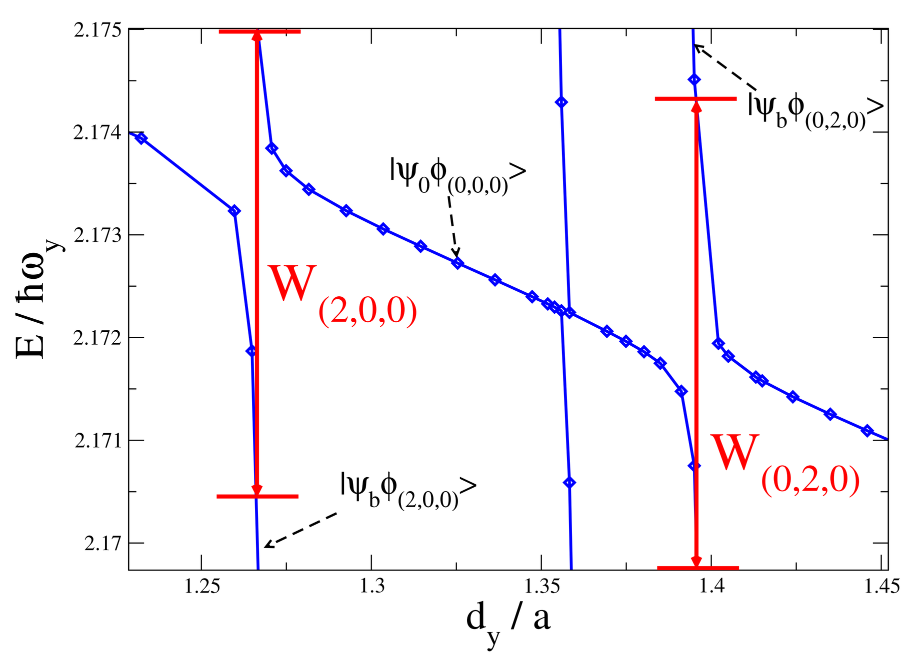


1. Bound state energy $E_b^{\text{REL}}(a)$ (using harmonic approximation)
2. Energy E_1 of the lowest trap state : (using constant approximation)
3. Center-of-mass excitation Δ_n (using 1st-order perturbation theory)

Crossings occur for:

$$E_b^{\text{REL}} = E_1^{\text{REL}} - \Delta_n$$

Matrix Elements and CIR Selection Rules in 1D:



$$W_n \approx \delta_{n_z,0}\Big[\delta_{n_y,0}\langle\phi_{n_x}\psi^{(b)}|W_x|\psi_1\phi_0\rangle + \delta_{n_x,0}\langle\phi_{n_y}\psi^{(b)}|W_y|\psi_1\phi_0\rangle\Big]$$

- Resonances only for **transversally** (tight direction: ω_x, or ω_y) excited bound states.
- $\mathbf{n} = (2,0,0)$ and $\mathbf{n} = (0,2,0)$ are dominant. $\mathbf{n} = (n_x, n_y, n_z)$
- Degenerate for $\omega_x = \omega_y$, **splitting** for $\omega_x \neq \omega_y$

Matrix Elements and CIR Selection Rules in 2D:

$$W_n \approx \delta_{n_y,0}\delta_{n_z,0}\langle\phi_{n_x}\psi^{(b)}|W_x|\psi_1\phi_0\rangle$$

- Resonances only for **transversally** (tight direction: ω_x) excited bound states.
- $\mathbf{n} = (2,0,0)$ is dominant.

Ab Initio Calculation

Hamiltonian (2 atoms in optical lattice (OL)):

$$H(\mathbf{r},\mathbf{R}) = H_{\text{REL}}(\mathbf{r}) + H_{\text{COM}}(\mathbf{R}) + W(\mathbf{r},\mathbf{R})$$
$$H_{\text{REL}}(\mathbf{r}) = T_{\text{REL}}(\mathbf{r}) + V_{\text{REL}}(\mathbf{r}) + U_{\text{int}}(r)$$
$$H_{\text{COM}}(\mathbf{R}) = T_{\text{COM}}(\mathbf{R}) + V_{\text{COM}}(\mathbf{R})$$

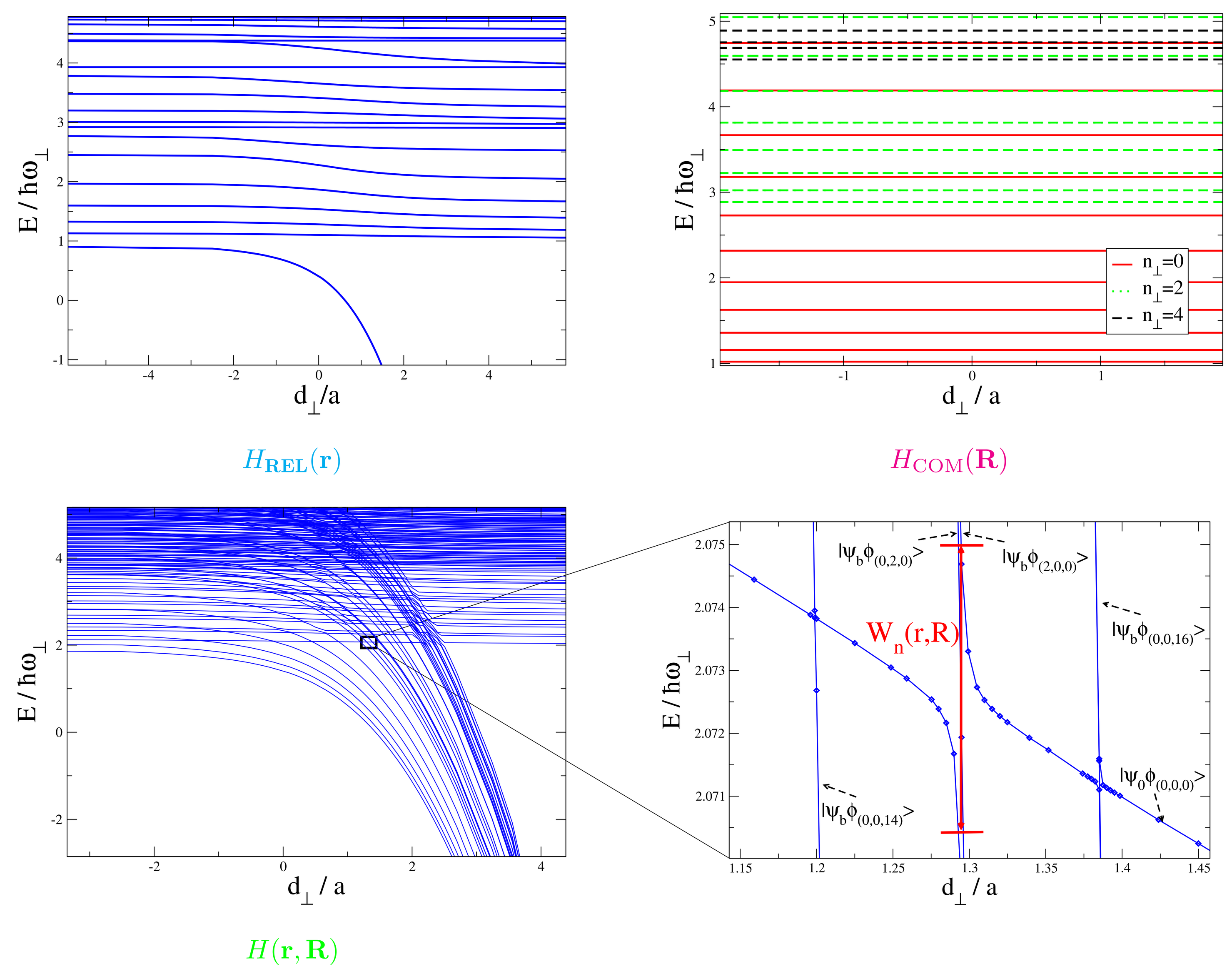
$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ relative motion (REL) coordinate
$\mathbf{R} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ center of mass motion (COM) coordinate
$T_{\text{REL}}(\mathbf{r})$, $T_{\text{COM}}(\mathbf{R})$ kin. energy operators
$V_{\text{REL}}(\mathbf{r})$, $V_{\text{COM}}(\mathbf{R})$ separable parts of OL potential
$W(\mathbf{r},\mathbf{R})$ non-separable OL terms
$U_{\text{int}}(r)$ interatomic interaction potential

Exact Diagonalization:

The energy spectrum of the full Hamiltonian is calculated. To include the COM-REL coupling the optical lattice (OL) potential is expanded in a Taylor series and truncated after the 6th power (sextic potential). The interatomic interaction is included using full Born-Oppenheimer (BO) potential curves. The scattering length is varied by an inner-wall manipulation of the BO curves.

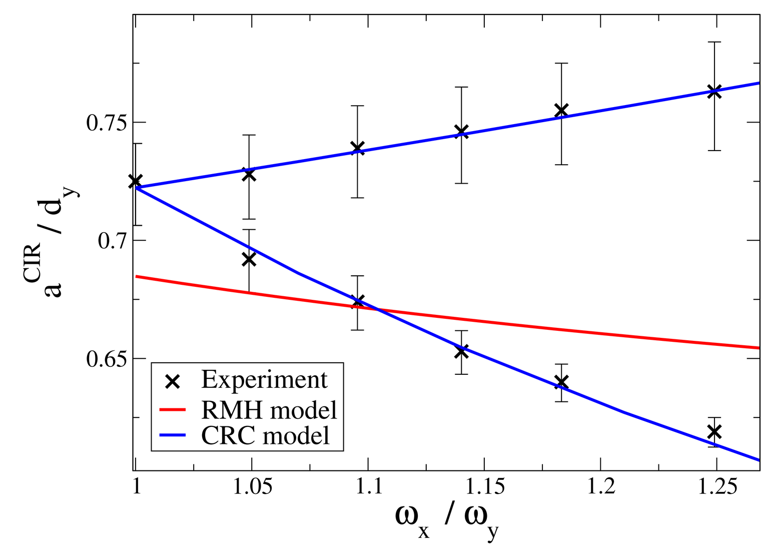
Composition of Energy Spectra:

Energy spectra for two atoms confined in a sextic trap with $\omega_x/\omega_z = \omega_y/\omega_z = 10$.



Comparison with Experiment

CIR Positions in Quasi 1D:



CIR Positions in Quasi 2D:

Method	$a^{(\text{CIR})}/d_y$
Experiment	0.593
CRC model	0.595
RMH model	*negative value*

Perfect agreement of inelastic CIR with loss resonances of experiment.

Conclusion

Coupling between center-of-mass and relative motion leads to inelastic confinement-induced resonances. The experiment of Haller et al. [7] measured inelastic losses rather than directly the effective interaction strength. Hence, it is much more sensitive to the here present inelastic CIRs than to the elastic ones predicted by RMH models [1,2]. As **perfect agreement** of the presented model with experiment [7] shows, **the measured loss resonances are inelastic CIR which occur because of COM-REL coupling due to the anharmonicity of the external confinement**. (For more details see arXiv:1104.1561 [8].)

References

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